

HW II , Math 530, Fall 2014

Ayman Badawi

QUESTION 1. (i) Let $(H, *)$, and (K, δ) be groups. Define a new group (F, ∇) (called external direct product of H and K), where $F = H \times K = \{(a, b) \mid a \in H, b \in K\}$. If $(a_1, b_1), (a_2, b_2) \in F$, then $(a_1, b_1)\nabla(a_2, b_2) = (a_1 * a_2, b_1 \delta b_2)$. It is clear that F is a group (trivial), do not prove that. Now let $D = H_1 \times K_1$ for some subgroup H_1 of H and some subgroup K_1 of K. Prove that D is a subgroup of F. Find two groups H, K such that $F = H \times K$ has a subgroup D, but $D \neq H_1 \times K_1$ for some subgroup H_1 of H and some subgroup K_1 of K.

(ii) Consider the symmetric-group on 5-gon (D_{10}, o) (see class notes). Calculate $F_2 O F_3$. What does R_4 mean? Find R_3^{-1}. Find $F_2 o R_3$ and $R_3^{-1} o F_2$. Can you generalize?

(iii) Let $(G, *)$ be a group and suppose that for some $a, b \in G$, we have $a * b = b * a$. Prove that $a * b^{-1} = b^{-1} * a$ and $a^{-1} * b^{-1} = b^{-1} * a^{-1}$.

(iv) Let F, H be subgroups of a group D.

a. Prove that $H \cap F$ is a subgroup of D.

b. Suppose $H \not\subseteq F$ and $F \not\subseteq H$. Prove $H \cup F$ is never a group (subgroup of G).

c. Assume $|H| = 10$ and $|F| = 4$. Prove that $H \cup F$ is never a group (subgroup of G).

Faculty information

Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.
E-mail: `abadawi@aus.edu, www.ayman-badawi.com`